Exercise 37

The table gives the height as time passes of a typical pine tree grown for lumber at a managed site.

Tree age (years)	14	21	28	35	42	49
Height (feet)	41	54	64	72	78	83

Source: Arkansas Forestry Commission

If $H(t)$ is the height of the tree after t years, construct a table of estimated values for H' and sketch its graph.

Solution

$H'(t)$ is the rate at which the height of the tree is increasing with respect to time (units of feet/year). To obtain the values of $H'(t)$, calculate the slope of the secant line going through two adjacent t values. At $t = 14$, for example,

$$H'(t) = \frac{H(21) - H(14)}{21 - 14} = \frac{54 - 41}{7} \approx 1.86.$$

At $t = 21$, there are two secant lines.

$$H'(t) = \frac{H(21) - H(14)}{21 - 14} = \frac{54 - 41}{7} = \frac{13}{7} \approx 1.86$$

$$H'(t) = \frac{H(28) - H(21)}{28 - 21} = \frac{64 - 54}{7} = \frac{10}{7} \approx 1.43$$

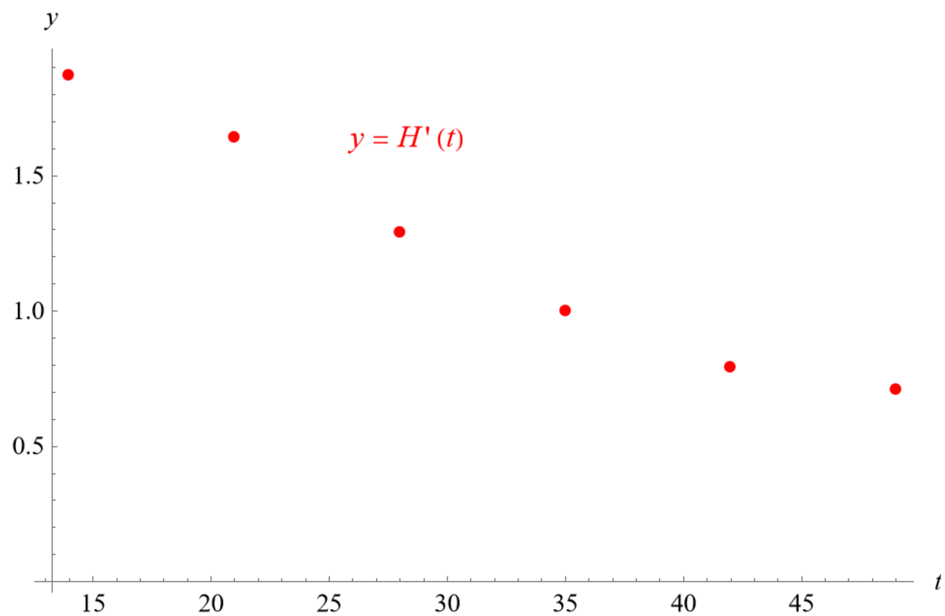
At such times where there are two possible secant lines, take the average for the best estimate.

$$\frac{\frac{13}{7} + \frac{10}{7}}{2} = \frac{23}{14} \approx 1.64$$

Below is a table of estimated values for $H'(t)$.

t	$H(t)$	$H'(t)$
14	41	1.86
21	54	1.64
28	64	1.29
35	72	1.00
42	78	0.79
49	83	0.71

Below is a graph of H' versus t.



It shows that a typical pine tree grows slower as it reaches maturity.